

# The Einstein-Cartan-Elko system

C. G. Böhrer\*

*ASGBG/CIU, Department of Mathematics, Apartado Postal C-600,  
University of Zacatecas (UAZ), Zacatecas, Zac 98060, Mexico*

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## Abstract

The present paper analyses the Einstein-Cartan theory of gravitation with Elko spinors as sources of curvature and torsion. After minimally coupling the Elko spinors to torsion, the spin angular momentum tensor is derived and its structure is discussed. It shows a much richer structure than the Dirac analogue and hence it is demonstrated that spin one half particles do not necessarily yield only an axial vector torsion component. Moreover, it is argued that the presence of Elko spinors partially solves the problem of minimally coupling Maxwell fields to Einstein-Cartan theory.

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\*Electronic address: boehmer@hep.itp.tuwien.ac.at

## I. INTRODUCTION

Einstein-Cartan theory is probably the simplest and straightest generalisation of Einstein's theory of general relativity. It is based on the usual Einstein-Hilbert action, however, it is not assumed that torsion vanishes, as was originally done by Einstein. If in the Einstein-Hilbert action the metric and the torsion are considered as independent variables, then the variations with respect to them yield two field equations. The first one relates the (non-symmetric) Einstein tensor to the canonical energy-momentum tensor, whereas the second one relates torsion to the spin angular momentum of matter, see e.g. [1, 2, 3, 4, 5, 6].

In the usual Einstein gravity, matter (mass) couples to the curvature of spacetime, whereas spin does not couple to geometrical quantities. The Poincaré Lie algebra can however be classified by the values of the two Casimir operators, see e.g. [7], mass squared  $M^2$  and angular momentum squared  $S^2 = s(s+1)$ , where  $s$  is the spin taking the usual values  $0, \pm 1/2, \dots$ . Therefore it seems quite natural [4, 6] to consider a theory of gravitation that takes both quantities, i.e. *mass and spin*, into account. The minimal coupling of standard model matter fields has been thoroughly discussed in [4] and the physical consequences were analysed. Ever since it allegedly became well known that spin one half matter fields only couple to the axial vector part of the torsion tensor, and that higher spin massive particles couple also to other parts of the torsion tensor [8]. It turns out, however, that the spin one half Elko spinors do couple to all parts of the torsion tensor, although they are spin one half. To understand these new properties from a physical point of view, some facts about the recently discovered Elko spinors should be recalled [9], with a condensed version published in [10]. The Elko spinors belong to a wider class of so-called flagpole spinors, see [11] for the Lounesto spinor field classification. The term Elko spinors originates from the German *Eigen*spinoren des *Ladungskonjugations*operator (eigenspinors of the charge conjugation operator).

Elko spinors are based on the eigenspinors of the charge conjugation operator, are non-standard Wigner class spinors and therefore yield a non-local field theory. They obey the unusual property  $(CPT)^2 = -\mathbb{1}$ . More explicitly, an Elko spinor is defined by [9, 10]

$$\lambda = \begin{pmatrix} \pm \sigma_2 \phi_L^* \\ \phi_L \end{pmatrix}, \quad (1)$$

where  $\phi_L^*$  denotes the complex conjugate of  $\phi_L$  and  $\sigma_2$  denotes the second Pauli matrix.

The upper sign stands for the self conjugate spinor and the lower sign for the anti self conjugate spinor with respect to the charge conjugation operator. The same physical content would have been obtained, had one started with the right-handed two-spinor  $\phi_R$ . For all details regarding the field theory of Elko spinors I refer the reader to the two fundamental papers [9, 10]. The two two-spinors  $\sigma_2\phi_L^*$  and  $\phi_L$  have opposite helicities and hence should be distinguishable. To do so, one writes

$$\lambda_{\{-,+\}} = \begin{pmatrix} \pm\sigma_2\phi_L^{+*} \\ \phi_L^+ \end{pmatrix}, \quad \lambda_{\{+,-\}} = \begin{pmatrix} \pm\sigma_2\phi_L^{-*} \\ \phi_L^- \end{pmatrix}, \quad (2)$$

where the first entry of the helicity subscript  $\{-, +\}$  refers to the upper two-spinor and the second the lower two-spinor. This subscript is henceforth denoted by indices  $u, v, \dots$ . Since the Elko spinors with respect to the standard Dirac dual  $\bar{\psi} = \psi^\dagger \gamma^0$  have an imaginary bi-orthogonal norm [9, 10], a new dual has to be defined so that a consistent field theory emerges. This Elko dual is given by

$$\bar{\lambda}_u = i \varepsilon_u^v \lambda_v^\dagger \gamma^0, \quad (3)$$

with the skew-symmetric symbol  $\varepsilon_{\{+,-\}}^{\{-,+\}} = -1 = -\varepsilon_{\{-,+\}}^{\{+,-\}}$  (note, that for Dirac spinors the term  $i \varepsilon_u^v$  is just  $\delta_u^v$ ). With the dual defined above one finds (by construction) the standard relation

$$\bar{\lambda}_u(\mathbf{p}) \lambda_v(\mathbf{p}) = \pm 2m \delta_{uv}, \quad (4)$$

where  $\mathbf{p}$  denotes the momentum.

As it has been pointed out above, the Elko spinors have a double helicity structure, opposed to Dirac spinors, where both two-spinors have the same helicity. The key feature of the Einstein-Cartan theory is, that spin couples to certain parts of geometry, i.e. torsion. Therefore, it is clear that this double helicity structure is much more sensitive to torsion than in the analogue Dirac case. Hence it is also expected (and shown in Section III) that the minimal coupling of Elko spinors to torsion yields a more interesting torsion tensor than in the Dirac spinor case.

The paper is organised as follows. In Section II the introduction of spinors into spacetimes is recalled. Section III describes general aspects of the Einstein-Cartan-Elko system and in Section IV gauge couplings in Einstein-Cartan theory are discussed. Conclusions of this work are presented in the final Section V.

## II. EINSTEIN-CARTAN THEORY AND SPINORS

The main aim of this section is to shortly recall some features of the Einstein-Cartan theory of gravitation, with a particular focus on the anholonomic formulation. If the starting point is a special relativistic field theory, one can apply the “comma to semicolon” rule ( $\partial_a \rightarrow \nabla_a$ ) to its Lagrangian in order to find the minimally coupled spacetime field theory.

Torsion is most naturally taken into account by assuming the existence of a covariant derivative operator  $\tilde{\nabla}_a$  that is not torsion free [1, 2, 3, 4, 5]. Quantities denoted with a tilde always take torsion into account. Therefore the minimal scheme to introduce torsion can symbolically be written as  $\nabla_a \rightarrow \tilde{\nabla}_a$  and the complete path from a special relativistic to an Einstein-Cartan field theory can be formulated as

$$\partial_a \rightarrow \nabla_a \rightarrow \tilde{\nabla}_a \quad (5)$$

The latter non-torsion free covariant derivative can be split according to

$$\tilde{\nabla}_a \lambda = \partial_a \lambda - \frac{1}{4} \Gamma_{abc} \gamma^b \gamma^c \lambda + \frac{1}{4} K_{abc} \gamma^b \gamma^c \lambda, \quad (6)$$

where  $K_{abc}$  is the contortion (and not contorsion) tensor,

$$\tilde{\Gamma}_{bc}^a = \Gamma_{bc}^a - K_{bc}^a. \quad (7)$$

The skew-symmetric part of the connection defines the torsion tensor  $S_{bc}^a$  to be

$$S_{bc}^a = \tilde{\Gamma}_{[bc]}^a = \frac{1}{2} (\tilde{\Gamma}_{bc}^a - \tilde{\Gamma}_{cb}^a). \quad (8)$$

By virtue of the last two equations, torsion and contortion are related by

$$S_{bc}^a = \frac{1}{2} (K_{cb}^a - K_{bc}^a), \quad (9)$$

Denoting the torsion free covariant derivative by  $\nabla_a$ , the covariant derivative, when acting on a spinor (6) can be written in the following form

$$\tilde{\nabla}_a \lambda = \nabla_a \lambda + \frac{1}{4} K_{abc} \gamma^b \gamma^c \lambda. \quad (10)$$

Since the covariant derivative obeys the Leibnitz rule, and the action of  $\tilde{\nabla}_a$  on the scalar  $\bar{\lambda} \lambda$  is known, for the dual Elko spinor  $\bar{\lambda}$  (see equation (3)) one therefore finds

$$\tilde{\nabla}_a \bar{\lambda} = \nabla_a \bar{\lambda} - \frac{1}{4} K_{abc} \bar{\lambda} \gamma^b \gamma^c. \quad (11)$$

After having defined, how to introduce a classical field theory into the Einstein-Cartan theory, one can now formulate the action. Since the Einstein-Cartan Lagrangian is simply the Einstein-Hilbert action (with metric and torsion regarded as independent variables), the coupled field equations can be derived from the total action given by

$$S = \int \left( \frac{1}{2\kappa} \tilde{R} + \tilde{\mathcal{L}}_{\text{mat}} \right) \sqrt{-g} d^4x, \quad (12)$$

where the speed of light was set to one ( $c = 1$ ). The Ricci scalar is denoted by  $\tilde{R}$  (computed from the complete connection with the contortion contributions),  $g$  is the determinant of the metric,  $\tilde{\mathcal{L}}_{\text{mat}}$  the (minimally coupled) matter Lagrangian and  $\kappa = 8\pi G$  is the coupling constant. The field equations of Einstein-Cartan theory [4] are obtained by varying the total action function (12) with respect to the metric  $\eta^{ij}$  and the contortion  $K^i_{jk}$  (or torsion) as independent variables, which yields

$$\tilde{R}_{ij} - \frac{1}{2} \tilde{R} \eta_{ij} = \kappa \Sigma_{ij}, \quad (13)$$

$$S^{ij}_{\phantom{ij}k} + \delta^i_k S^{jl} - \delta^j_k S^{il} = \kappa \tau^{ij}_k. \quad (14)$$

In equation (14)  $\tau^{ij}_k$  is the spin angular momentum tensor and  $\Sigma_{ij}$  in (13) is the total (or canonical) energy-momentum tensor which is not symmetric. It is defined by

$$\Sigma_{ij} = \sigma_{ij} + (\tilde{\nabla}_k + K_{lk}^l)(\tau_{ij}^k - \tau_j^k{}_i + \tau^k{}_{ij}), \quad (15)$$

where  $\sigma_{ij}$  is the metric energy-momentum tensor given by the variation of the matter Lagrangian with respect to the metric. It should be emphasised that the trajectories of test particles are obtained by integrating the conservation equations of the energy and the angular momentum, see e.g. [4], and in general are neither geodesics nor autoparallels. It is also important to note that the field equations (14) are algebraic equations for the torsion. Therefore  $\tau_{ijk} = 0$  immediately implies the vanishing of the torsion. Hence torsion is only present in spacetime regions with torsion sources and therefore it is not propagating. Because of the algebraic torsion equation one can formally eliminate the torsion by its spin sources in the action. This yields one effective or combined field equation

$$G_{ij} = \kappa \hat{\sigma}_{ij}, \quad (16)$$

where the energy-momentum tensor on the right-hand side is given by the following rather complicated expression [4]

$$\hat{\sigma}_{ij} = \sigma_{ij} + \kappa \left( -4\tau_{i[l}^k \tau_{j\phantom{ij}k]}^l - 2\tau_i^{kl} \tau_{jkl} + \tau^{kl}{}_i \tau_{klj} + \frac{1}{2} \eta_{ij} (4\tau_m^k{}_{[l} \tau^{ml}{}_{k]} + \tau^{klm} \tau_{klm}) \right). \quad (17)$$

Note that the square of the coupling constant  $\kappa^2$  enters the torsion contributions in  $\hat{\sigma}_{ij}$  in equation (17). This effective or combined energy-momentum tensor is symmetric and satisfies the usual conservation equation  $\nabla^j \hat{\sigma}_{ij} = 0$ . After recalling some basic principles of Einstein-Cartan theory, the minimal coupling of Elko spinors to Einstein-Cartan theory is now considered in the next section.

### III. ELKO SPINORS IN EINSTEIN-CARTAN THEORY

The Elko spinors obey scalar field-like equations of motion since their mass dimension is one. The Elko Lagrangian constructed in [9] reads

$$\mathcal{L}_{\text{Elko}} = \eta^{ab} \partial_a \bar{\lambda} \partial_b \lambda - m^2 \bar{\lambda} \lambda + \alpha [\bar{\lambda} \lambda]^2. \quad (18)$$

As described above, one can apply the scheme (5) to the above Lagrangian and arrives at

$$\tilde{\mathcal{L}}_{\text{Elko}} = \eta^{ab} \tilde{\nabla}_a \bar{\lambda} \tilde{\nabla}_b \lambda - m^2 \bar{\lambda} \lambda + \alpha [\bar{\lambda} \lambda]^2. \quad (19)$$

One should be slightly careful with the Lagrangian (19), since if varied with respect to the metric, the resulting term  $\nabla_a \bar{\lambda} \nabla_b \lambda$  is not necessarily symmetric, even in spacetimes without torsion. Hence one should add the Elko conjugate equation so that consistent field equations emerge. Hence the correct covariant Elko Lagrangian reads

$$\tilde{\mathcal{L}}_{\text{Elko}} = \eta^{ab} \frac{1}{2} \left( \tilde{\nabla}_a \bar{\lambda} \tilde{\nabla}_b \lambda + \tilde{\nabla}_b \bar{\lambda} \tilde{\nabla}_a \lambda \right) - m^2 \bar{\lambda} \lambda + \alpha [\bar{\lambda} \lambda]^2. \quad (20)$$

It should be noted the the Lagrangian of complex massive scalars is formally very similar to the Elko Lagrangian. The difference, which is in fact crucial, is that the Elkos are spinors. The covariant derivative, when acting on scalars, is just the partial derivative  $\tilde{\nabla}_a \phi = \partial_a \phi$ . Hence, scalar particles (spin 0) cannot couple minimally to the torsion of spacetime [4], however, higher spin matter can.

By taking into account equations (10) and (11) one can split the Elko Lagrangian into its torsion free part and the torsion contributions and arrives at

$$\tilde{\mathcal{L}}_{\text{Elko}} = \mathcal{L}_{\text{Elko}} + \frac{1}{4} K^a{}_{bc} \nabla_a \bar{\lambda} \gamma^b \gamma^c \lambda - \frac{1}{4} K^a{}_{bc} \bar{\lambda} \gamma^b \gamma^c \nabla_a \lambda - \frac{1}{16} K^a{}_{bc} K_{ade} \bar{\lambda} \gamma^b \gamma^c \gamma^d \gamma^e \lambda. \quad (21)$$

Variation with respect to the contortion tensor  $K^i{}_{jk}$  yields the spin angular momentum tensor

$$\tau^{kj}{}_i = \frac{\delta \tilde{\mathcal{L}}_{\text{Elko}}}{\delta K^i{}_{jk}} = \frac{1}{4} \tilde{\nabla}_i \bar{\lambda} \gamma^j \gamma^k \lambda - \frac{1}{4} \bar{\lambda} \gamma^j \gamma^k \tilde{\nabla}_i \lambda, \quad (22)$$

which equivalently can be written as follows

$$\tau^{kj}{}_i = \frac{1}{4}\nabla_i\bar{\lambda}\gamma^j\gamma^k\lambda - \frac{1}{4}\bar{\lambda}\gamma^j\gamma^k\nabla_i\lambda - \frac{1}{16}K_{iab}\bar{\lambda}\gamma^j\gamma^k\gamma^a\gamma^b\lambda - \frac{1}{16}K_{iab}\bar{\lambda}\gamma^a\gamma^b\gamma^j\gamma^k\lambda. \quad (23)$$

An important feature of this spin angular momentum tensor is that it depends on the contortion (or torsion) itself, and it is obviously not totally skew-symmetric (and cannot be expressed entirely as an axial torsion vector). This fact should indeed be emphasised, Elko spinors are possible sources for all irreducible parts of the torsion tensor. In fact, many references on Einstein-Cartan theory state that it is well known that spin one half particles yield a axial torsion vector. However, as was just shown, this statement is not completely correct. The non-standard Wigner class Elko spinors (that are spin one half) lead to more torsion structure than the Dirac spinors. Henceforth, the above statement should be formulated more carefully.

In order to be complete, the Elko Lagrangian (19) implies the following metric energy-momentum tensor

$$\sigma_{ij} = \frac{1}{2}(\tilde{\nabla}_i\bar{\lambda}\tilde{\nabla}_j\lambda + \tilde{\nabla}_j\bar{\lambda}\tilde{\nabla}_i\lambda) - \eta_{ij}\tilde{\mathcal{L}}_{\text{elko}}. \quad (24)$$

In order to value the particular form of the spin angular momentum tensor (23) for Elko spinors, its form should be compared with those from other field theories. For sake of completeness, let us note again, that for scalar fields (spin 0) one has

$$\tau_{ijk} = 0, \quad (25)$$

while the spin angular momentum tensor of Dirac fermions (spin 1/2) is given by

$$\tau^{ijk} = \tau^{[ijk]} = \frac{1}{4}\bar{\psi}\gamma^{[i}\gamma^j\gamma^{k]}\psi, \quad (26)$$

which is totally skew-symmetric and has only four independent components [4]. The next logical field to consider would be the Maxwell field, i.e. the massless spin one gauge field. However, gauge fields cannot be minimally coupled to torsion since the resulting terms containing the contortion (or torsion) turn out not to be gauge invariant [4]. This subtle problem is avoided by considering massive spin one fields. However, by assuming a particular form of the torsion tensor and a modified gauge transformation, a minimal coupling scheme could still be developed in [12]. The same holds for any Yang-Mills type field theory, the minimal coupled scheme would yield terms that brake gauge invariance.

In the introduction it was argued that the presence of two Casimir operators, mass and spin, serves as an argument why any theory of gravity should take both into account. Consequently, Yang-Mills type field theories should also be treated using the minimal coupling scheme. From my point of view, to exclude gauge fields is as unnatural as excluding spin from any theory of gravitation. This issue certainly requires further investigation.

Let us come back to the spin angular momentum tensor of massive spin one fields [4], which reads

$$\tau_{kji} = \frac{1}{2} \left( U_k \tilde{\nabla}_{[j} U_{i]} - U_j \tilde{\nabla}_{[k} U_{i]} \right) = \frac{1}{2} \left( U_k \nabla_{[j} U_{i]} - U_j \nabla_{[k} U_{i]} \right) + S_{i[j}{}^l U_{k]} U_l. \quad (27)$$

In that latter case the spin angular momentum tensor also depends on the contortion, like in the Elko spinor case.

By repeated application of  $\gamma^a \gamma^b = 2\eta^{ab} - \gamma^b \gamma^a$  one can change the order of the  $\gamma$ -matrices in the third term of the spin angular momentum tensor (23),  $\gamma^j \gamma^k \gamma^a \gamma^b \rightarrow \gamma^a \gamma^b \gamma^j \gamma^k$ , so that the last term appears twice and additional terms are also present. Hence the spin angular momentum tensor reads

$$\begin{aligned} \tau^{kj}{}_i = & \frac{1}{4} \nabla_i \bar{\lambda} \gamma^j \gamma^k \lambda - \frac{1}{4} \bar{\lambda} \gamma^j \gamma^k \nabla_i \lambda - \frac{1}{8} K_{iab} \bar{\lambda} \gamma^a \gamma^b \gamma^j \gamma^k \lambda \\ & + \frac{1}{2} K_i{}^{jk} \bar{\lambda} \lambda + \frac{1}{4} K_{ia}{}^j \bar{\lambda} \gamma^a \gamma^k \lambda - \frac{1}{4} K_{ia}{}^k \bar{\lambda} \gamma^a \gamma^j \lambda. \end{aligned} \quad (28)$$

With the help of equation (9) one can rewrite the second field equation (14) in terms of the contortion tensor

$$K^{ji}{}_k - K^{ij}{}_k + \delta_k^i K_l{}^{jl} - \delta_k^j K_l{}^{il} = 2\kappa \tau^{ij}{}_k \quad (29)$$

Inserting in the latter equation  $\tau^{ij}{}_k$  as given by (28) leads to

$$\begin{aligned} K^{ji}{}_k - K^{ij}{}_k + \delta_k^i K_l{}^{jl} - \delta_k^j K_l{}^{il} = & k \left( \frac{1}{2} \nabla_k \bar{\lambda} \gamma^j \gamma^i \lambda - \frac{1}{2} \bar{\lambda} \gamma^j \gamma^i \nabla_k \lambda \right. \\ & \left. - \frac{1}{4} K_{kab} \bar{\lambda} \gamma^a \gamma^b \gamma^j \gamma^i \lambda + K_k{}^{ji} \bar{\lambda} \lambda + \frac{1}{2} K_{ka}{}^j \bar{\lambda} \gamma^a \gamma^i \lambda - \frac{1}{2} K_{ka}{}^i \bar{\lambda} \gamma^a \gamma^j \lambda \right). \end{aligned} \quad (30)$$

The latter equation is an algebraic relation for the 24 component of the contortion tensor, that can in principle be solved  $K = K(\bar{\lambda}, \lambda, \nabla \bar{\lambda}, \nabla \lambda)$ . The Elko spinors appear quadratically in the contortion tensor  $K_{bc}{}^a$ . Because of the algebraic equation of motion for contortion one can eliminate the contortion tensor in the Elko Lagrangian. Since the last term in the Elko Lagrangian (21) is quadratic in the contortion tensor this yields an effectively sextic



Elko self-interaction term. This is quite similar to what happens in the Dirac spinor case. In that case the Lagrangian is linear in the contortion, and the equations of motion yield the contortion quadratically in the spinors so that the Lagrangian, after eliminating the contortion, contains a quartic self-interaction term.

One could furthermore decompose the contortion tensor into its irreducible part, the contortion vector  $K_b = K_{ab}{}^a$ , the axial vector  $A_a = \varepsilon_{abcd}K^{bcd}$  and a tensor  $Q_{abc}$ , say, containing the remaining contortion parts. In case of Dirac spinors, see (26), this turns out to be quite useful. For Elko spinors however, this is not very enlightening, since all components essentially enter equation (30).

#### IV. ELKO SPINORS AND GAUGE COUPLINGS

As was already pointed out in the previous section, gauge fields cannot be consistently coupled to Einstein-Cartan theory, since the resulting term would spoil gauge invariance. Ways out have been proposed, e.g. in Ref. [12] by assuming a rather special form of the torsion tensor. In what follows it will be argued that the existence of Elko spinors in nature might partially solve this problem. The dominant interaction of Elko spinors [9] are with gravity (19) and through the Higgs doublet with the following interaction Lagrangian

$$\mathcal{L}_{\text{int}} = c_H \Phi^\dagger \Phi \bar{\lambda} \lambda, \quad (31)$$

where  $\Phi$  denotes the Higgs doublet. However, one can also construct an interaction Lagrangian where the Elko spinor couples to an Abelian gauge field [9] with field strength  $F_{ab}$  that takes the form

$$\mathcal{L}_{\text{int}} = c_F \bar{\lambda} [\gamma^a, \gamma^b] F_{ab} \lambda. \quad (32)$$

Such an interaction yields effective mass terms for the photon, so that the coupling constant  $c_F$  is heavily constrained by experimental data on the photon mass, see e.g. [13, 14]. It should also be emphasised that the Elko spinors are neutral with respect to  $U(1)$  and  $SU(n)$ .

Nonetheless, such an interaction, though certainly very small, would be present in nature if also the Elko spinors would turn out to be more than just a theoretical construct. Consequently, the photons would become massive and the resulting field theory of a massive spin one field could consistently be coupled to Einstein-Cartan theory. Unfortunately, this

possibility does not contain a general scheme to couple gauge fields minimally to Einstein-Cartan theory. The simplest interaction term of Elko spinors with Yang-Mills fields<sup>1</sup> would be of the form

$$\mathcal{L}_{\text{int}} = c_{\text{FF}} \text{Tr}(F_{ab}^{\mathcal{A}} F_{\mathcal{A}}^{ab}) \bar{\lambda} \lambda, \quad (33)$$

where with  $\mathcal{A}$  the internal  $SU(n)$  group index was denoted. As in the above mentioned case, the coupling constant  $c_{\text{FF}}$  must be extremely small.

## V. SUMMARY AND CONCLUSIONS

The dual helicity structure of Elko spinors motivated the study of Elko spinors in spacetimes with torsion. In contrast to the Dirac spinors, the Elko spinors' spin angular momentum tensor shows a much richer structure. Physically this can be explained from the mentioned dual helicity structure, whereas from a mathematical point of view it is a consequence of the scalar-like Lagrangian with spinorial fields.

The Elko spinors are prime first principle candidates for dark matter. It was shown that the presence of Elko spinors would necessarily leads to massive photons. Hence, the resulting massive spin one field theory could be coupled consistently to the Einstein-Cartan theory.

In a series of papers [15, 16, 17], the extended (with torsion) spin-coefficient formalism was developed by Griffiths and Jodia. It would be promising to apply this extended formalism to the Einstein-Cartan-Dirac system so that the geometrical structure induced by Elko spinors in spacetimes with torsion can be analysed more invariantly.

In order to further study the physics of the Einstein-Cartan-Elko system, cosmological investigations seem to be the most natural starting point, since the number of degrees of freedom is greatly reduced. On the other hand, gravity theories with torsion have also been elaborately studied by several authors, see e.g. [18], assuming the spacetime to be homogeneous and isotropic. If the cosmological principle is applied [19], the torsion (or contortion) tensor is restricted to the following non-vanishing components, a vector and an axial vector torsion part. Therefore, since all technical ingredients are now known, the analysis of the cosmological Einstein-Cartan-Elko can in principle begin and will be the

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<sup>1</sup> I thank Daniel Grumiller for elucidating this point.

subject of further investigation. One of the most interesting issues will certainly be the study of torsion effects [20, 21, 22, 23] in the inflationary epoch, however, driven by Elko spinors.

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